UNIT 3

Review

Use the following polynomial to solve problems 1–4.

$2x^4 + 14x^3 - x - 7$

1. Leading term: $2x^4$

2. Constant term: -7

3. Coefficient of the cubic term: 14

	x	7
$2x^3$	$2x^4$	$14x^3$
-1	$-x$	-7

4. Factor the polynomial by grouping. State if each factor is a monomial, a binomial, or a trinomial.

$(2x^3-1)(x+7)$ binomials

Factor each polynomial completely. Look for identities and expanded binomials.

5. $8a^3 + 1$ $((2a)^3 + 1^3)$

$= (2a+1)(4a^2 - 4a^2 + 1)$

6. $a^4 - 81b^4$ $(a^2+9b^2)(a^2-9b^2)$

$= (a^2+9b^2)(a+3b)(a-3b)$

7. $18x^3 - 32x$ $2x(9x^2-16)$

$= 2x(3x+4)(3x-4)$

8. $x^3y + 3x^2y^2 + 3xy^3 + xy^4$

$xy(x^2+3xy+3y^2+y^3)$

Choose the best answer.

9. Which of the following shows all possible rational roots of the equation $5x^5 - 7x^3 - 9 = 0$? $\frac{P}{Q} = \frac{\pm 1 \pm 3 \pm 9}{\pm 1 \pm 5}$

A. $\{\pm 1, \pm 5\}$

B. $\{\pm 1, \pm 3, \pm 9\}$

C. $\left\{\pm\frac{5}{9}, \pm 1, \pm\frac{5}{3}, \pm 3, \pm 5, \pm 9\right\}$

D. $\left\{\pm\frac{1}{5}, \pm\frac{3}{5}, \pm 1, \pm\frac{9}{5}, \pm 3, \pm 9\right\}$

10. Which is a factor of $2x^3 - x^2 - 13x - 6$?

A. $(x - 3)$

B. $(x - 2)$

C. $(x + 1)$

D. $(x + 4)$

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & -6 \\ & \downarrow & 6 & 15 & 6 \\ \hline & 2 & 5 & 2 & 0 \end{array}$$

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For each of the following systems of equations, the first equation is graphed. Graph the second equation to find the solution(s) of each system.

11. $y = -x^3 - x^2 - 1$

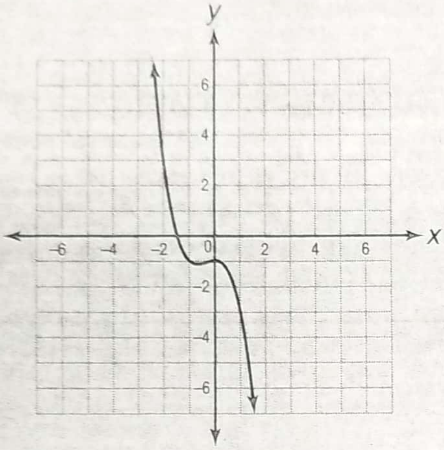
$y = x^2 - 4$

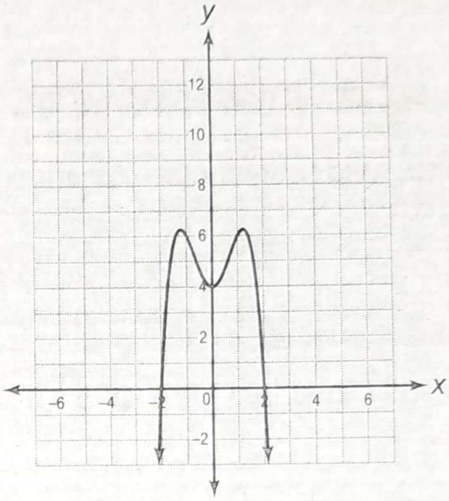
solution(s): ____________________

12. $y = -x^4 + 3x^2 + 4$

$y = -6x + 12$

solution(s): ____________________





Choose the best answer.

13. Which of the following statements about the function $g(x) = -3x^4 + 3$ is **not** true?

$g(-x) = -3(-x)^4 + 3 = -3x^4 + 3$

A. It is an odd function.

B. It has a *y*-intercept at (0, 3).

C. As *x* approaches $-\infty$, $g(x)$ approaches $-\infty$.

D. As *x* approaches ∞, $g(x)$ approaches $-\infty$.

14. Which of the following statements about the function $f(x) = 0.5x^3$ is **not** true?

$f(-x) = 0.5(-x)^3 = -0.5x^3$

A. It is an odd function.

B. It has an *x*-intercept at (0, 0).

C. As *x* approaches $-\infty$, $f(x)$ approaches ∞.

D. As *x* approaches ∞, $f(x)$ approaches ∞.

Identify the number of roots and the possible number(s) of positive and negative roots of each of the following polynomials.

15. $3x^2 - 7x + 2$

$+ \quad - \quad +$

$3(-x)^2 - 7(-x) + 2$

$-3x^2 + 7x + 2$

number of roots: 2

positive roots: 2 or 0

negative roots: 1

16. $x^4 - 6x^3 + 5x^2 + x - 3$

$+ \quad - \quad + \quad + \quad -$

$(-x)^4 - 6(-x)^3 + 5(-x)^2 + (-x) - 3$

$x^4 + 6x^3 + 5x^2 - x - 3$

number of roots: 4

positive roots: 3 or 1

negative roots: 1

Use the polynomial function $f(x) = x^3 + 2x^2 + 3x + 6$ to solve problems 17–19.

	x	2
x^2	x^3	$2x^2$
3	$3x$	6

17. Factor to identify the zero(s) and x-intercept(s) of the function. Justify your response.

$(x+2)(x^2+3) = 0$

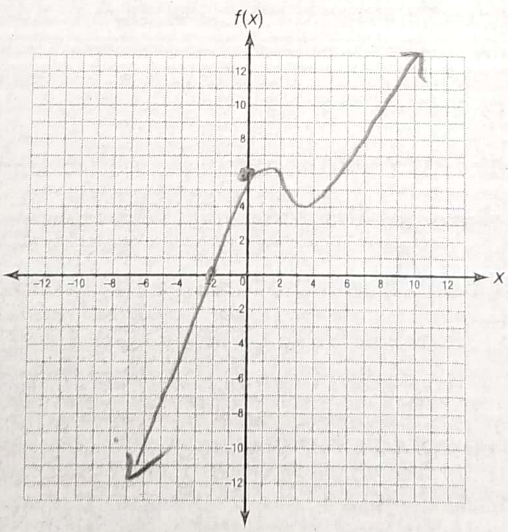
$x = -2, \pm 3i$

18. Use the equation to describe the end behavior of the function.

odd - positive $\quad x \to -\infty,\ y \to -\infty$

$x \to +\infty,\ y \to +\infty$

19. Sketch a graph of the function. Explain how you determined additional points on the graph.



y-intercept is where $x = 0$

Solve.

20. A polynomial function has a leading term of $23x^6$ and is an even function. Describe the following characteristics of the graph. If it is not possible to know from the information given, write *unknown*.

symmetry: Symmetric about the y axis

end behavior: both ends going to $+\infty$

zeros: $x=0$ with multiplicity of 6

Derive the sum and difference identities of polynomials of degree 6, called hexics, to solve problems 21 and 22.

21. **PROVE** The identity for the difference of two hexics, $x^6 - y^6$, is shown below. Rewrite $x^6 - y^6$ as the difference of two cubes to prove that the identity is true.

$$x^6 - y^6 = (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$$

22. **COMPARE** Rewrite $x^6 + y^6$ as the sum of two cubes, then derive an identity to represent the sum of two hexics.

$x^6 + y^6 =$ ____________

How does the identity for the sum of hexics differ from the identity for the difference of hexics?

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