Completing the Square

Solve $ax^2 + bx + c = 0,\ a \neq 0$ *by completing the square*

Ex. $x^2 - 12x + 20 = 0$

Step 1

If $a \neq 1$, *divide both sides of the equation by a*

$x^2 - 12x + 20 = 0$

$-20 \quad -20$

Step 2

Rewrite the equation so that the constant term is alone on one side of the equal sign.

leave a space

$x^2 - 12x + 36 = -20 + 36$

$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

Step 3

Square half of coefficient of x term and add this square to both sides of the equation.

$x^2 - 12x + 36 = -20 + 36$

$(x-6)(x-6) = 16$

Step 4

Factor the LHS and simplify the RHS.

Use the square root method to solve for x.

$\sqrt{(x-6)^2} = \sqrt{16}$

$x - 6 = \pm 4$

$x - 6 = 4 \qquad x - 6 = -4$

$+6 \quad +6 \qquad +6 \quad +6$

$x = 10 \qquad x = 2$

Ex. $\frac{2x^2}{2} - \frac{24x}{2} - \frac{178}{2} = \frac{-4}{2}$

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$x^2 - 12x - 89 = -2$

$+89 \quad +89$

$x^2 - 12x + 36 = 87 + 36$

$\left(\frac{-12}{2}\right)^2$

$= (-6)^2$

$= 36$

$\sqrt{(x-6)^2} = \sqrt{123}$

$x - 6 = \pm\sqrt{123}$

$x - 6 = \sqrt{123} \qquad x - 6 = -\sqrt{123}$

$+6 \quad +6 \qquad +6 \quad +6$

$x = \sqrt{123} + 6,\ -\sqrt{123} + 6$

$Ex.\ x^2 - 4x - 14 = 0$